

Study on Muscle Fatigue Measurement During Elbow Flexion-Extension Using EMG and OpenSim

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Abstract

This paper proposes a method of surface electromyography (sEMG)-based mensuration of muscle fatigue during elbow flexion-extension. The proposed method has two parts. First, the joint angle and locations of body segments are estimated using inertial measurement units (IMU) and the OpenSim biomechanics simulator. Second, muscle fatigue is measured using OpenSim and a three-compartment muscle fatigue model. The experimental results show that muscle fatigue increases in accordance with the momentum.

Keywords

Muscle Fatigue; EMG; IMU, Biomechanics; Opensim, Motion Capture; Elbow Flexion-Extension

Introduction

Commonly used surface electromyography (sEMG)-based methods for measuring muscle fatigue use the mean power frequency or median frequency of the electromyogram (EMG) recorded while a subject performs static contraction. However, using these methods, it is difficult to accurately measure muscle fatigue in daily activities, exercise, and labor activities that actually represent a subject's motion that may cause or aggravate musculoskeletal diseases. To solve this problem, some previous studies have additionally used the a post-processing step based on pattern recognition techniques (Rogers and MacIsaac 2011, 2010).

In this paper, we propose a muscle fatigue measurement method based on kinematic information. This method connects muscle fatigue to motion, and can obtain biomechanics values, such as the muscle force and muscle activation, and moment from the user's movements. To measure the motion from which kinematic information is derived, marker-based motion capture (MbMoCap) devices have been commonly used. However these are very expensive, difficult to setup, and have space constraints. To overcome these limitations, we propose a method to use inertial measurement units (IMU). These are inexpensive, easy to use, and can be used outdoors (Daniel Roetenberg 2013).

The OpenSim biomechanics simulator (OpenSim, Stanford Univ., USA) and three-compartment muscle fatigue model (Xia and Frey Law 2008) were used to calculate muscle fatigue. Because input form or extension of OpenSim is only MbMoCap-based, marker positions were estimated using the IMU and input to the inverse kinematics in OpenSim to calculate the joint angle. This joint angle was input to the static optimization in OpenSim to calculate muscle activation. Then, the coefficients of the EMG-muscle activation transformation were calculated using the muscle activation and EMG to transform the afterward measured EMG into muscle activation. Finally, the EMG-based muscle activation was input to the muscle fatigue model to calculate muscle fatigue which is dimensionless. This flow chart is shown in Fig. 1.

Mensuration of muscle fatigue during elbow flexion-extension (EFE), which has 1 degree of freedom (DOF), was developed.

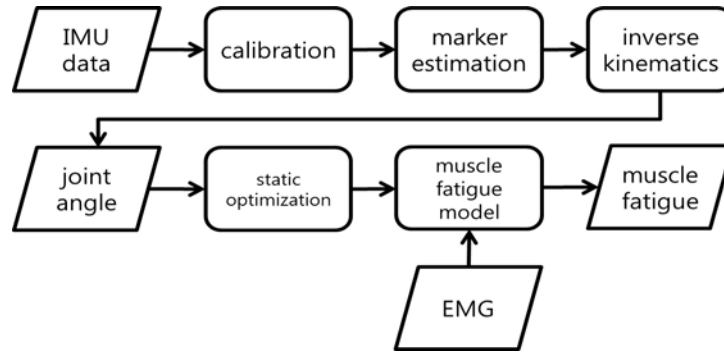


FIG. 1 OVERALL PROCESS OF THE PROPOSED MUSCLE FATIGUE MEASUREMENT METHOD.

Joint Angle Estimation Using IMU and Opensim

In this section, we present the calculation of the joint angles between the upper arm and lower arm using the position information of body segments obtained from the IMU data and the inverse kinematics provided from OpenSim. For the OpenSim body model, Arm26 was used (Holzbaur, Murray, and Delp 2005). This model has 1-DOF hinge joints at the shoulder and elbow joints, three segments—the thorax, humerus, and forearm—and six muscles. However in our experiments, only three muscles—the long head of the biceps brachii (BIClong), the short head of the biceps brachii (BICshort), and the brachialis (BRA) —were measured. The marker set was created referring to the Heidelberg Upper Extremity model (Rettig et al. 2009). The marker set has ten markers: the jugular notch (CLAV), xiphoid process (STRN), C7, T10, acromion (SHO), tuberositas deltoidea humeri (HUM), lateral epicondyle (LAT), olecranon (OLC), ulna styloid process (ULN), and radial styloid process (RAD). The marker positions are shown in Fig. 2.

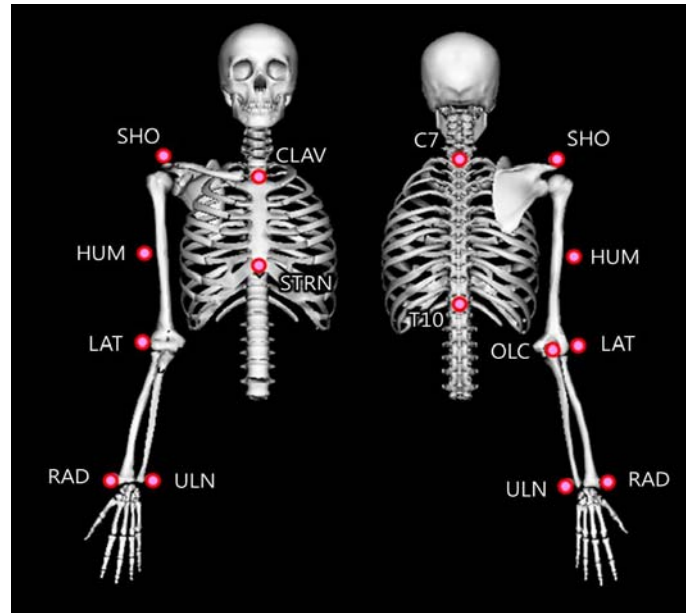


FIG. 2 LOCATION OF THE MARKERS ATTACHED TO THE UPPER ARM AND TORSO.

Mbmocap Data Calibration

The body segments of the model were scaled based on MbMoCap. In addition, the coordinates of the measured data should be calibrated. First, the MbMoCap data was calibrated. The calibration equation is as follows:

$${}^{default}p_i = {}^{default}T_{exp} * {}^{exp}p_i \quad (1)$$

where ${}^{default}p_i$ is the i th marker position in OpenSim, ${}^{exp}p_i$ is the i th measured marker position, and ${}^{default}T_{exp}$ is the transformation matrix that calibrates the coordinates from the experimental coordinates to the coordinates of

OpenSim, calculated by the least-squares method (LSM) (Sorkine 2009).

$${}_{\text{exp}}^{\text{default}}T = \underset{i=\text{marker}}{\operatorname{argmin}} \sum w_i \left\| {}_{\text{exp}}^{\text{default}}T * {}_{\text{exp}}^{\text{exp}}p_i - {}_{\text{default}}^{\text{default}}p_i \right\|^2 \quad (2)$$

where w_i is weight, which is greater than the others if the corresponding marker is on a bony place or placed at the center of the body. The marker positions in OpenSim were calculated using the geometry of the scaled model. The transformation matrix was calculated using the MbMoCap data measured under a static pose similar to the default in OpenSim (default pose). Finally, all measured marker positions were pre-multiplied by the transformation matrix.

IMU Data Calibration

The IMU data were calibrated using MbMoCap. If IMU-based motion capture is accurate as MbMoCap—, in other words, the rotational differences between MbMoCap and the IMU are constant—the calibrator is,

$${}_{\text{calibrator}}q_j = {}_{\text{MbMoCap}}q_j * {}_{\text{IMU}}q_j^{-1} \quad (3)$$

where ${}_{\text{calibrator}}q_j$ is the MbMoCap-IMU calibrator of the j th joint, ${}_{\text{MbMoCap}}q_j$ is the quaternion of the j th joint measured with MbMoCap, and ${}_{\text{IMU}}q_j^{-1}$ is the inverse quaternion of the j th joint measured with IMU. The rotational differences between MbMoCap, IMU, and calibrator are shown in Fig. 3. First, the rotational difference from the default pose to MbMoCap was calculated using the LSM for each body segment.

$${}_{\text{default}}^{\text{exp}}R_j = \underset{i=\text{marker}}{\operatorname{argmin}} \sum \left\| {}_{\text{default}}^{\text{exp}}R_j * {}_{\text{default}}^{\text{default}}p_i - {}_{\text{exp}}^{\text{exp}}p_i \right\|^2 \quad (4)$$

where ${}_{\text{default}}^{\text{exp}}R_j$ is the rotation matrix of the j th joint from the default pose to MbMoCap data, ${}_{\text{default}}^{\text{default}}p_i$ is the i th default pose marker position in the local coordinate, and ${}_{\text{exp}}^{\text{exp}}p_i$ is the i th measured marker position in the local coordinate. ${}_{\text{default}}^{\text{exp}}R_j$ was transformed into ${}_{\text{MbMoCap}}q_j$ for convenient calculation. After the calibration stage, the calibrator for each joint was pre-multiplied for each of IMU data.

$${}_{\text{calibrated}}q_j(t) = {}_{\text{calibrator}}q_j * {}_{\text{IMU}}q_j(t) \quad (5)$$

The marker positions were estimated as follows:

$${}_{\text{IMU}}p_i(t) = {}_{\text{calibrated}}q_j(t) * {}_{\text{default}}p_i * {}_{\text{calibrated}}q_j(t)^{-1} + T_j(t) \quad (6)$$

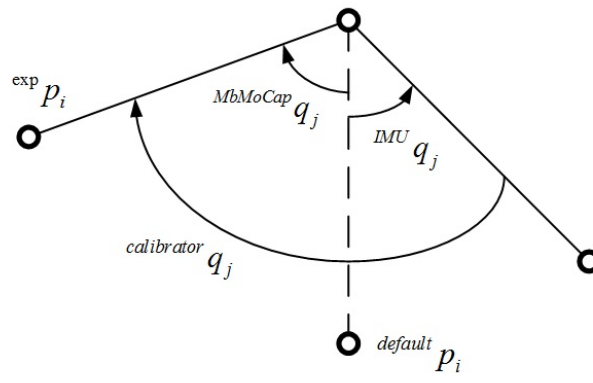


FIG. 3 ROTATION OF JOINT ANGLE FOR EACH MBMOCAP, IMU, AND CALIBRATOR.

where $T_j(t)$ is the translation vector of the j th joint in time t , and the vector was added to each of corresponding markers to tune the child joint to the parent joint. Finally, a *.trc file the input of OpenSim was created using these

marker positions to calculate the joint angle using OpenSim.

Muscle Fatigue Measurement

Muscle fatigue was calculated using the joint angles and EMG. Muscle activation was calculated by inputting the joint angle into static optimization in OpenSim. Using this muscle activation and EMGs, the coefficients of the EMG–muscle activation transformation were calculated. Then, the afterward measured EMG was transformed into muscle activation. The EMG-based muscle activation was input into three-compartment muscle fatigue model (Xia and Frey Law 2008), and muscle fatigue is calculated last.

EMG–Muscle Activation Transformation

The EMG–muscle activation transformation was derived using EMG–neural activation and neural activation–muscle activation transformation. The transfer function of the EMG–neural activation is as follows (Buchanan et al. 2004, David G. Lloyd 2002, Manal et al. 2002):

$$\frac{u(t)}{e(t)} = \frac{\alpha z^{-d}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2}} = \frac{\alpha}{1 + \beta_1 z^{-1} + \beta_2 z^{-2}} \quad (7)$$

where $u(t)$ is neural activation; $e(t)$ is EMG; and α , β_1 , and β_2 are coefficients conditioned as follows:

$$\alpha - \beta_1 - \beta_2 = 1, \beta_1 = \gamma_1 + \gamma_2 \beta_2 = \gamma_1 * \gamma_2, |\gamma_1| < 1, \quad |\gamma_2| < 1 \quad (8)$$

Where d is delay time and is equal to 0.04. The simplified transformation of neural–muscle activation is as follows (Buchanan et al. 2004, David G. Lloyd 2002, Manal et al. 2002):

$$a(t) = \frac{e^{A * u(t)} - 1}{e^A - 1} \quad (9)$$

where $a(t)$ is muscle activation and A is a non-linear shape factor. The coefficients γ_1 , γ_2 , and A differ from the other muscles and are valid until the sEMG sensor is removed (Criswell 2011). The coefficients were determined by minimizing the gap between the EMG and motion-based muscle activation with simulated annealing (Sartori et al. 2012).

$$\gamma_1, \gamma_2, A = \operatorname{argmin} \sum_{m=\text{muscle}} \sum_t \left\| {}^{EMG}a_m(t) - {}^{motion}a_m(t) \right\| \quad (10)$$

The initial values of the coefficients are $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, and $A = -0.1$ (Buchanan et al. 2004). After calculating the coefficients, the afterward measured EMG was transformed into muscle activation without the motion based information.

Muscle Fatigue Calculation

Muscle fatigue was calculated using a three-compartment fatigue model (Xia and Frey Law 2008).

$$RC(t) = M_A + M_R = 1 - M_F \quad (11)$$

where RC is residual capacity; M_A is the activated compartment; M_R is the resting compartment; and M_F is the fatigued compartment, which denotes muscle fatigue. The values of the compartments ranged from 0 to 1. As a muscle activates, the fatigued compartment increases, and the others decrease. Fig. 4 shows the muscle fatigue model. M_A , M_R , and M_F are calculated using the following equation:

$$\frac{dM_R}{dt} = -C(t) + R * M_F(t) \quad \frac{dM_A}{dt} = C(t) - F * M_A(t) \quad \frac{dM_F}{dt} = F * M_A(t) - R * M_F(t) \quad (12)$$

where $C(t)$ is the activation–deactivation driving controller, F is the fatigue coefficient, and R is the recovery coefficient. $C(t)$ is the same as shown in the following equation (Miguel T. Silva 2011):

$$C(t) = \begin{cases} L_R * (TL(t) - M_A) & (TL(t) \leq M_A) \\ L_D * (TL(t) - M_A) & (M_A < TL(t) < (M_A + M_R)) \\ L_D * M_R & ((M_A + M_R) < TL(t)) \end{cases} \quad (13)$$

where L_D is the force development factor; L_R is the relaxation factor; and TL is the target load which is (Xia and Frey Law 2008):

$$TL = \begin{cases} RC * BE & (TL \leq BE) \\ 1 & (BE < TL) \end{cases} \quad (14)$$

where BE is the voluntary brain effort. As M_F increases, RC decreases, and BE increases. Due to substituting $a(t)$ into the TL , the result of the simulation was the same as that of Xia's (Xia and Frey Law 2008). Therefore, $C(t)$ can be rewritten as,

$$C(t) = \begin{cases} L_R * (a(t) - M_A) & (a(t) \leq M_A) \\ L_D * (a(t) - M_A) & (M_A < a(t) < (M_A + M_R)) \\ L_D * M_R & ((M_A + M_R) < a(t)) \end{cases} \quad (15)$$

It is postulated that all muscle fibers are slow muscle fibers, so F is 0.01, R is 0.002, and L_R and L_D are 10 (Pereira 2009). Also, M_R is 1 before the muscle activates.

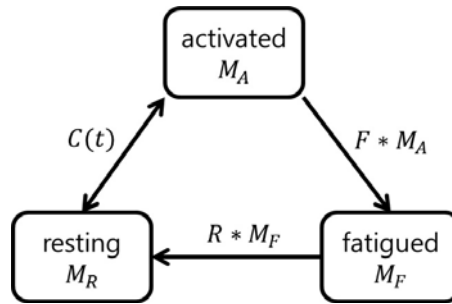


FIG. 4 THREE-COMPARTMENT FATIGUE MODEL.

Experimental Results

Database and Sensor Devices

A non-disabled male subject (27 years old, 181 cm, weighing 72 kg) was tested. Markers, IMUs, and the sEMG sensor were attached to the subject's body. Quaternion data were received from two wireless IMUs (50 Hz) (EBIMU24GV2, e2box, South Korea). A kalman filter-based embedded algorithm calculated the quaternion, and the error of the IMU was known to be within 1° in daily life wrist motions (I. W. Jung 2014). Because only the start and end times could be recorded, it was assumed that the data were recorded equidistantly. Lost data were interpolated with spherical linear interpolation (Slerp) (Eberly 2010). Retro-reflective markers placed on the subject's body were recorded (60 Hz) using eight motion capture system cameras (Motion Analysis, Motion Analysis, USA). The time data of the IMUs and MbMoCap were synchronized using the LSM after measurement. A dry and bi-polar type sEMG sensor was used (2000 Hz) (Delsys Trigno, Delsys, USA). The received signal was 20–450 Hz band-pass filtered to remove baseline noise and artifact noise, full wave rectified, normalized by maximum voluntary contraction (MVC), and 6Hz low pass filtered (Sartori et al. 2012).

Comparative Experiments for Joint Angle Estimation

The accuracy of the joint angle using IMUs was tested by comparing the results with the MbMoCap-based joint angle. EFE was performed 10 times under 5 different frequencies (0.2, 0.25, 0.33, 0.5, and 1 Hz [1, 2, 3, 4, and 5 s for each cycle]). The results were measured to study the correlation between the error and angular velocity. IMUs were

attached to the pronator tuberosity and the nearby HUM, and the marker set described previously was attached. The corresponding markers and IMUs for each segment are provided in Table. 1.

TABLE. 1 LOCATION OF MARKERS AND NUMBER OF IMUS ATTACHED TO EACH BODY SEGMENT

segment	IMU	marker
thorax	none	CLAV, STRN, C7, T10, SHO
humerus	#1	HUM, LAT
forearm	#2	OLC, ULN, RAD

At the calibration stage, the default pose was measured. After the calibration stage, full range of motion (ROM) (0~140°) EFE was measured. Whole steps were measured 10 times.

One of the the examples of the IMU and marker-based joint angle under 1 Hz frequency is the same as shown in Fig. 5. The solid blue line is the IMU-based joint angle, and the black dotted line is the marker-based joint angle. The errors increase largely after the subject starts to move—even elbow joint angle exceeded the ROM sometimes. Errors were not recovered after the subject finished moving. Because the method used in this study is not used to improve accuracy but instead converts IMU data to MbMoCap data, this might be caused by the inner side of the IMU-like hardware and embedded algorithm.

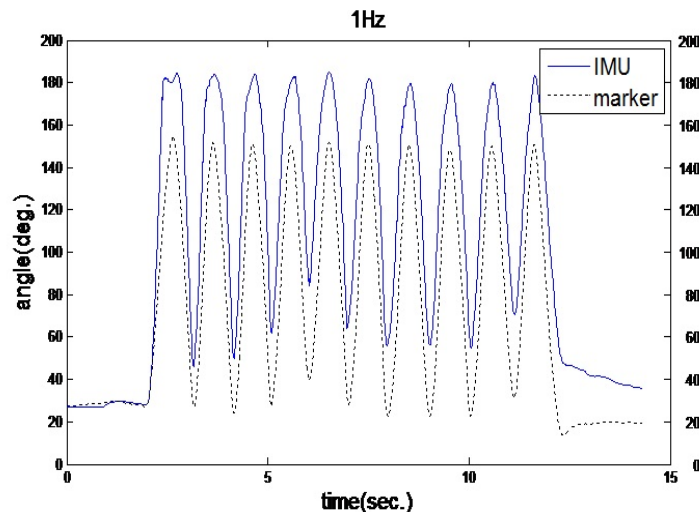


FIG. 5 COMPARISON OF EFE JOINT ANGLE ESTIMATION RESULTS AT 1HZ (BLUE LINE: IMU, BLACK DOTTED LINE: MBMOCAP).

TABLE. 2 MEAN ABSOLUTE ERRORS BETWEEN JOINT ANGLES ESTIMATED BY IMU AND MBMOCAP

Error Hz	Mean absolute error (deg.)	
	Mean	Standard deviation
0.2	47.35	13.42
0.25	44.3	7.31
0.33	41.36	8.75
0.5	39.22	8.23
1	36.55	8.84

The mean absolute errors of the mean and standard deviation for each frequency are given in Table. 2. The mean absolute error under different frequencies using the t-test shows that there are no significant differences under different frequencies. A large motion or large acceleration which instantaneously occurs might generate an

irreversible error which would not be reduced even after a subject stops the motion.

Muscle Fatigue Measurement Results

To measure and test muscle fatigue, a three step measurement was performed. First, to normalize EMG, the MVC of the EMG was measured while making the forearm perpendicular to the humerus and keeping the humerus vertical. Second, to calculate coefficients of the EMG—muscle activation transformation, a full ROM EFE with a 3-kg dumbbell was measured for 30 s while keeping the humerus vertical. Third, the same exercise was measured, but to determine the correlation between momentum and muscle fatigue, the subject performed EFE 10, 20, and 30 times. The subject rested between measurements, and whole steps were measured ten times.

An example of Muscle fatigue for the three compartments in BIClong is shown in Fig. 6-8. The black line is M_A ; the blue line is M_R ; and the red line is M_F , which denotes muscle fatigue. The mean and standard deviation (SD) of the accumulated fatigue for each muscle are the same as given in Table. 3. The first column means performed EFE times. As the subject increased the number of EFEs in a measurement, fatigue increased linearly. Meanwhile, as the total times in a measurement increased, fatigue increased geometrically. The accumulated fatigue under different EFE times using the t-test shows that the differences between times are valid.

Because simulated annealing was used, the coefficients of the EMG—muscle activation transformation changed every time calculating with the same motion and EMG data. Therefore, the muscle activation and muscle fatigue also changed.

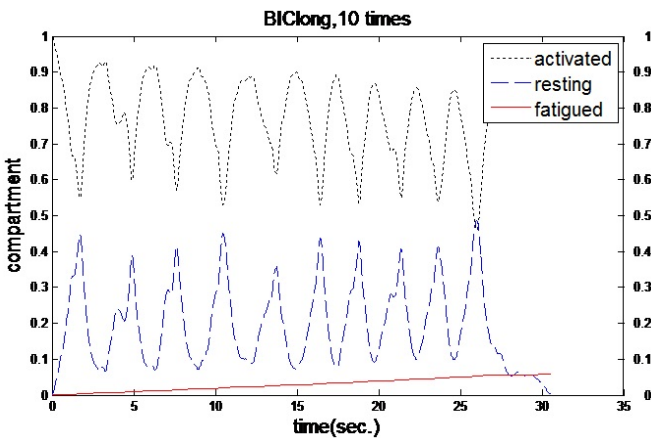


FIG. 6 MUSCLE FATIGUE MEASURED IN BICLONG UNDER 10 TIMES REPETITION.

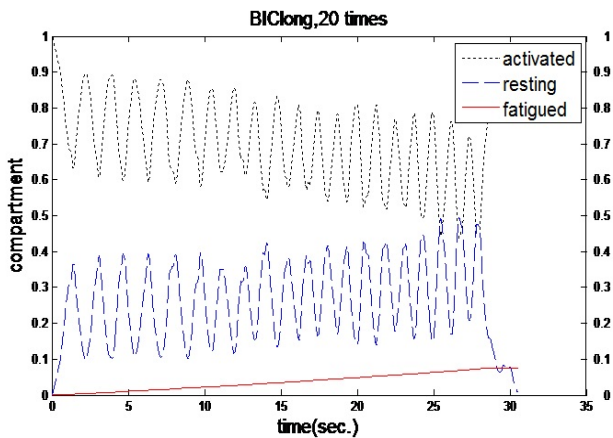


FIG. 7 MUSCLE FATIGUE MEASURED IN BICLONG UNDER 20 TIMES REPETITION.
UNDER 30 TIMES REPETITION.

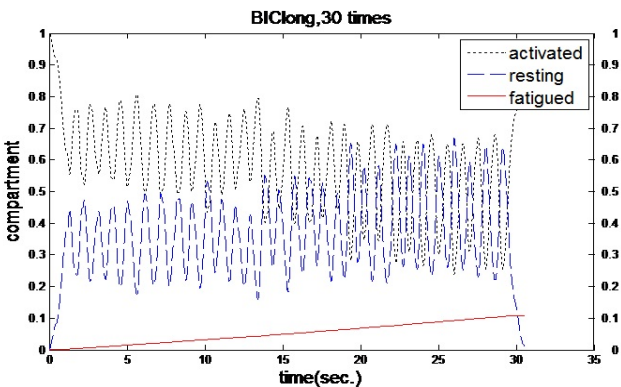


FIG. 8 MUSCLE FATIGUE MEASURED IN BICLONG

TABLE. 3 MEAN AND STANDARD DEVIATION OF ACCUMULATED MUSCLE FATIGUE FOR BICLONG, BICSHORT, AND BRA

Times	BIClong		BICshort		BRA	
	mean	SD	mean	SD	mean	SD
10	0.0637	0.0012	0.0381	0.0006	0.0318	0.0013
20	0.0836	0.0028	0.0523	0.0008	0.0466	0.0007
30	0.1204	0.0033	0.0826	0.0021	0.0743	0.0016

Discussion

To improve this study, first, IMU-based biomechanics simulator should be developed. This can reduce the calculation burden, avoiding several stages that are needed to estimate the marker positions and create the

MbMoCap-based input file described in this paper. Second, the EMG–muscle activation coefficients should be calculated using the moment instead of the muscle activation (Sartori et al. 2012). If forward dynamics is simulated with muscle activation-based coefficients, no solution is found or large errors are generated often. Third, to expand the DOF or measure more joint motions, the mensuration of spaculo–humeral rhythm using IMUs should be developed. This rhythm cannot be neglected but is very difficult to measure or model (Edward K. Chadwick 2014). If an IMU is attached to the sternum, which is the only sensor in the shoulder–torso segment, it is underdetermined to measure the spaculo-humeral rhythm. It might be impossible to measure this shoulder motion in several years because the current IMU has low sensitivity to translational and rotational small changes. Fourth, the muscle fatigue model should be tested and adjusted. It was developed recently and has been not verified yet (Ackermann 2007). The coefficients should be tested and adjusted first.

This study does have limitation. First, deeply positioned muscles such as the psoas and iliacus, which cannot be measured with sEMG, create a gap between measured and EMG-driven motion (Sartori et al. 2012). Second, because the proposed method is based on peripheral muscle models, central nerve system fatigue is neglected (Xia and Frey Law 2008).

Conclusion

In this study, muscle fatigue during EFE motion was measured using EMG and kinematic information. It can be used in not only static contraction, which is measured with conventional methods using mean power frequency or median frequency, but also dynamic motion. For convenient motion capturing, we proposed using IMUs which are easy to be used and less space restrictive in comparison with MbMoCap. The joint angles were calculated by estimating the marker positions with IMUs and OpenSim. In addition, EMG-based muscle fatigue was estimated using OpenSim and a three-compartment muscle fatigue model (Xia and Frey Law 2008). The experimental results showed that the error of the joint angle and the EFE frequency did not have a correlation, but that a large error is generated when the subject makes a large, instantaneous movement.

In future work, first, the accuracy of the joint angle estimation based on IMUs would be improved by developing hardware or algorithms to fulfil the requirements for biomechanics study. Second, IMU and sEMG based on BS would be developed to replace OpenSim. Third, to expand the DOF or more, the mensuration of spaculo–humeral rhythm using IMUs would be studied. Forth, the measurement of muscle fatigue should be verified or comparatively studied with a more objective experiment tools like force/torque sensors. Lastly, the proposed method should be tested on more subjects.

REFERENCES

- [1] Ackermann, Marko. 2007. "Dynamics and energetics of walking with prostheses." Institute of Engineering and Computational Mechanics, University of Stuttgart.
- [2] Buchanan, T. S., D. G. Lloyd, K. Manal, and T. F. Besier. 2004. "Neuromusculoskeletal modeling: estimation of muscle forces and joint moments and movements from measurements of neural command." *J Appl Biomech* 20 (4):367-95.
- [3] Criswell, Elenaor. 2011. *Cram's introduction to surface electromyography*. jones and bartlett publishers.
- [4] Daniel Roetenberg, Henk Luinge, and Per Slycke. 2013. "Xsens MVN: Full 6DOF Human Motion Tracking Using Miniature Inertial Sensors."
- [5] David G. Lloyd, Thor F. Besier. 2002. "An EMG-driven musculoskeletal model to estimate muscle forces and knee joint moments in vivo." *Journal of Biomechanics* 36:765-776.
- [6] Eberly, David. 2010. "Quaternion Algebra and Calculus." *Geometric Tools, LLC*.
- [7] Edward K. Chadwick, Dimitra Blana, Robert F. Kirsch, Antonie J. van den Bogert. 2014. "Real-Time Simulation of Three-Dimensional Shoulder Girdle and Arm Dynamics." *IEEE Trans Biomed Eng.* 7:1947-1956.
- [8] Holzbaur, K. R., W. M. Murray, and S. L. Delp. 2005. "A model of the upper extremity for simulating musculoskeletal surgery and analyzing neuromuscular control." *Ann Biomed Eng* 33 (6):829-40.

- [9] I. W. Jung, H. S. Park, Y. J. Lee, I. C. Youn. 2014. "Measurement of Wrist Joint Angles Using Intertial Sensor." International BioMedical Engineering Conference 2014, Gwangju Korea.
- [10] Manal, K., R. V. Gonzalez, D. G. Lloyd, and T. S. Buchanan. 2002. "A real-time EMG-driven virtual arm." *Computers in Biology and Medicine* 32 (1):25-36. doi: Doi 10.1016/S0010-4825(01)00024-5.
- [11] Miguel T. Silva, André F. Pereira, Jorge M. Martins. 2011. "An efficient muscle fatigue model for forward and inverse dynamic analysis of human movements." *Procedia IUTAM* 2 (2):262-274.
- [12] Pereira, André F. 2009. "Development of a Hill-Type Muscle Model With Fatigue for the Calculation of the Redundant Muscle Forces using Multibody Dynamics." The 1st Joint International Conference on Multibody System Dynamics, Lappeenranta, Finland.
- [13] Rettig, O., L. Fradet, P. Kasten, P. Raiss, and S. I. Wolf. 2009. "A new kinematic model of the upper extremity based on functional joint parameter determination for shoulder and elbow." *Gait Posture* 30 (4):469-76. doi: 10.1016/j.gaitpost.2009.07.111.
- [14] Rogers, D. R., and D. T. Macisaac. 2010. "Training a multivariable myoelectric mapping function to estimate fatigue." *J Electromyogr Kinesiol* 20 (5):953-60. doi: 10.1016/j.jelekin.2009.11.001.
- [15] Rogers, D. R., and D. T. MacIsaac. 2011. "EMG-based muscle fatigue assessment during dynamic contractions using principal component analysis." *Journal of Electromyography and Kinesiology* 21 (5):811-818. doi: DOI 10.1016/j.jelekin.2011.05.002.
- [16] Sartori, M., M. Reggiani, D. Farina, and D. G. Lloyd. 2012. "EMG-Driven Forward-Dynamic Estimation of Muscle Force and Joint Moment about Multiple Degrees of Freedom in the Human Lower Extremity." *Plos One* 7 (12). doi: ARTN e52618
- [17] DOI 10.1371/journal.pone.0052618.
- [18] Sorkine, Olga. 2009. "Least-Squares Rigid Motion Using SVD."
- [19] Xia, T., and L. A. Frey Law. 2008. "A theoretical approach for modeling peripheral muscle fatigue and recovery." *J Biomech* 41 (14):3046-52. doi: 10.1016/j.jbiomech.2008.07.013.